9/19/17 "Quality is not an act, it is a habit." -Aristotle

HW: "Infinite Limits and Limits at Infinity HW"
Test 1 on Tuesday 9/26

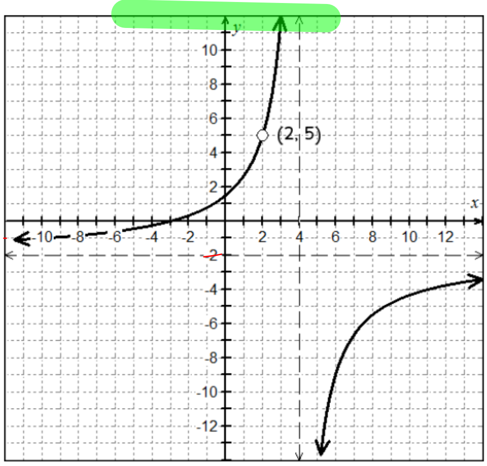
AIM: Limits at Infinity

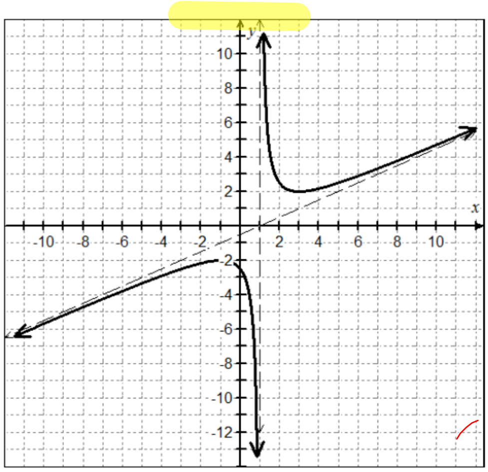
Infinite Limits and Limits at Infinity

In the lesson on <u>*Understanding Limits*</u> you were confronted with these two situations. Here we begin to **compare and contrast** the behavior of functions as they *approach infinity*, as well as, functions that *tend toward infinity* in certain circumstances. Let's go…

EX #1: Use the graphs of $f(x)$ and $g(x)$ shown below to compare and contrast behaviors involving infinity. Let's go...

$$f(x)=\frac{-2x^2-2x+12}{x^2-6x+8} \qquad g(x)=\frac{x^2-2x+5}{2x-2}$$





An Infinite Limit is:	A Limit at Infinity is:
lim as $x \to a$ that results in ∞ or $-\infty$	limit as $x \to \pm\infty$ "End Behavior"
$\lim\limits_{x\to 4^-} f(x) = \infty$	$\lim\limits_{x\to -\infty} f(x) = -2$
$\lim\limits_{x\to 4^+} f(x) = -\infty$	$\lim\limits_{x\to \infty} f(x) = -2$
$\lim\limits_{x\to 1^-} g(x) = -\infty$	$\lim\limits_{x\to -\infty} g(x) = -\infty$
$\lim\limits_{x\to 1^+} g(x) = \infty$	$\lim\limits_{x\to \infty} g(x) = \infty$
Discovery: Vertical asymptotes	Discovery: Horizontal and Oblique "End Behavior" Asymptotes

In Pre-Calculus you learned some basic truths about rational functions.

1. When a factor cancelled from the denominator a ___hole___ occurred.
2. When a factor would not cancel from the denominator a ___vertical asymptote___ occurred.

EX #2: Use the previous equations to find the limits analytically.

A. $\lim_{x \to 4^-} \frac{-2x^2 - 2x + 12}{x^2 - 6x + 8}$

Plug in 4 we get $\Rightarrow \frac{-28}{0}$

Plug in a value close 3.9

$$\frac{-2(3.9)^2 - 2(3.9) + 12}{(3.9)^2 - 6(3.9) + 8} = \frac{\text{negative}}{\text{negative}} = \text{Positive}$$

limit is ∞

B. $\lim\limits_{x \to 4^+} \dfrac{-2x^2 - 2x + 12}{x^2 - 6x + 8}$

C. $\lim\limits_{x \to 1^-} \dfrac{x^2 - 2x + 5}{2x - 2}$

D. $\lim\limits_{x \to 1^+} \dfrac{x^2 - 2x + 5}{2x - 2}$

Definition and Justification of Vertical Asymptotes

Case 1: $h(c)=\frac{non-zero}{zero}$ undefined	Case 2: $h(c)=\frac{zero}{zero}$ indeterminant
$x=c$ is: Vertical asymptote	$x=c$ is: hole
Limits could only be $\pm\infty$, DNE	

IN CALCULUS, YOU MUST USE NEW LANGUAGE IN ORDER TO JUSTIFY!

LIMIT DEFINITION (JUSTIFICATION) OF A VERTICAL ASYMPTOTE

If $\lim_{x \to a^-} f(x) = \pm\infty$

$x=a$ is a vertical asymptote

If $\lim_{x \to a^+} f(x) = \pm\infty$

$x=a$ is a vertical asymptote

"a" is a real number

EX #3: Use the function below to find any vertical asymptote(s) that exist. Justify your answer using limits.

$$h(x)=\frac{2x^2+9x-5}{x^2+3x-10}$$

Limits at Infinity

Next, we will explore **limits at infinity** in order to differentiate between the two conditions. Recall the lessons from Pre-Calculus related to analyzing the **end behavior of functions.** In the exercise below, use this prior knowledge to find each limit at infinity.

EX #4: Find each limit at infinity, explain your thinking.

A. $\lim_{x \to \infty} (x^2 - 4)(x^2 + 3)$

$\lim_{x \to \infty} (x^4 + 3x^2 - 4x^2 - 12)$

$\lim_{x \to \infty} x^4 - x^2 - 12$

Degree of 4

* If the degree is even then look at the sign of the lead coefficient

If its positive
limit as $x \to \pm\infty$ is ∞

If its negative
limit as $x \to \pm\infty$ is $-\infty$

B. $\lim_{x \to -\infty} (5x^3 - 2x + 4)$

If the degree is odd:

$x \to -\infty$ limit is $-\infty$ (lead coefficient is positive)
$x \to \infty$ limit is ∞

* If lead coefficient is negative:

$x \to -\infty$ limit is ∞
$x \to \infty$ limit is $-\infty$

C. $\lim_{x\to\infty} \frac{3x^2-4}{1x^2+1}$

Degree of numerator = Degree of denominator

$\lim = \frac{3}{1}$

Then there is a horizontal asymptote.

The limit = $\frac{\text{lead coefficient on TOP}}{\text{lead coefficient on BoTTom}}$

D. $\lim_{x\to-\infty} \frac{5x-2}{x^2+1}$

Degree of Numerator $<$ Degree of Denominator

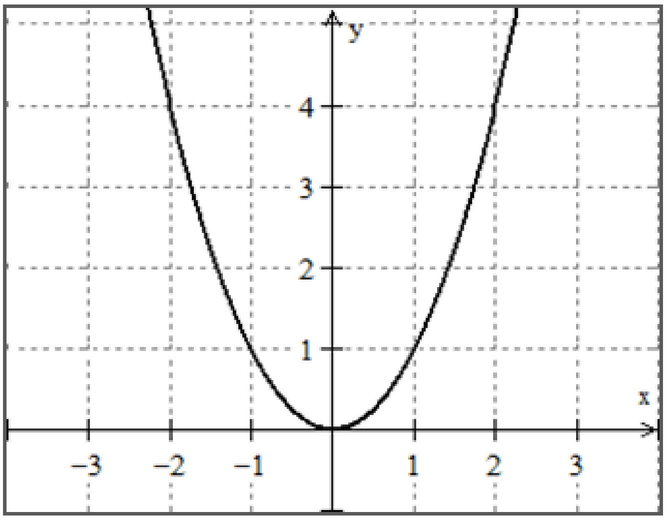
limit = 0

Horizontal asymptote is @ $y=0$

EX #5: There are only four possible outcomes when you explore behavior to the extreme right or left.

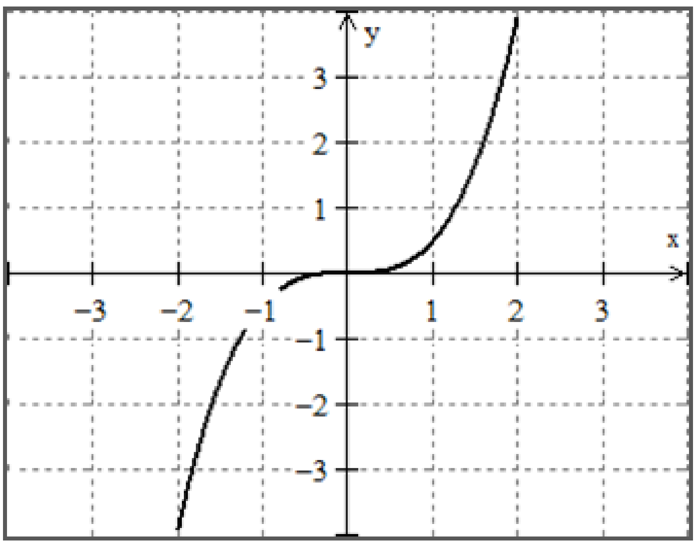
1. The curve can increase without bound.

$$\lim_{x \to \infty} f(x) = \infty$$



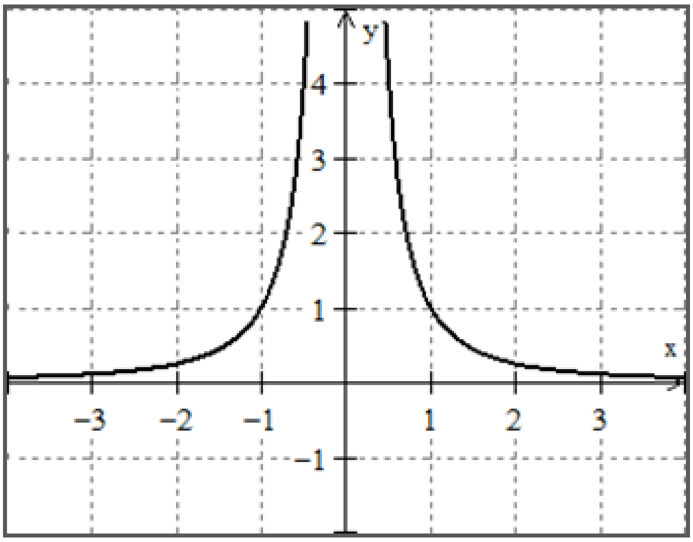
2. The curve can decrease without bound.

$$\lim_{x \to -\infty} f(x) = -\infty$$



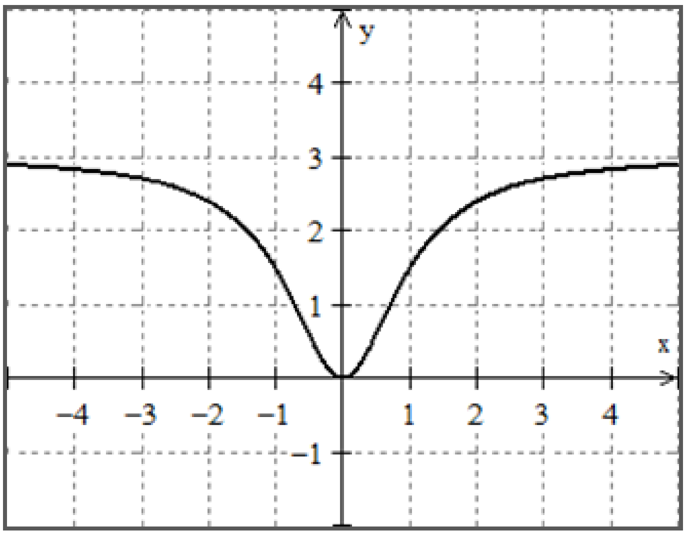
3. The curve can become asymptotic to the x-axis.

$$\lim_{x \to \infty} f(x) = 0$$



4. The curve can become asymptotic to a specific *y*-value.

$$\lim_{x \to -\infty} f(x) = 3$$



Revisiting the rules for finding potential horizontal asymptotes for rational functions from Pre-Calculus, you can use the idea of a limit and calculus to see why those rules hold true.

1. If degree of numerator is less than degree of denominator (bottom heavy), then limit is zero.

$$\lim_{x \to \pm\infty} f(x) = 0$$

2. If degree of numerator equals degree of denominator (powers equal), then limit is the ratio of coefficients of the highest degree.

$$\lim_{x \to \pm\infty} f(x) = \frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$$

3. If degree of numerator is greater than degree of denominator (top heavy), then limit does not exist.

$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

$$\lim_{x \to \infty} \frac{x^2}{x+1} = \infty$$

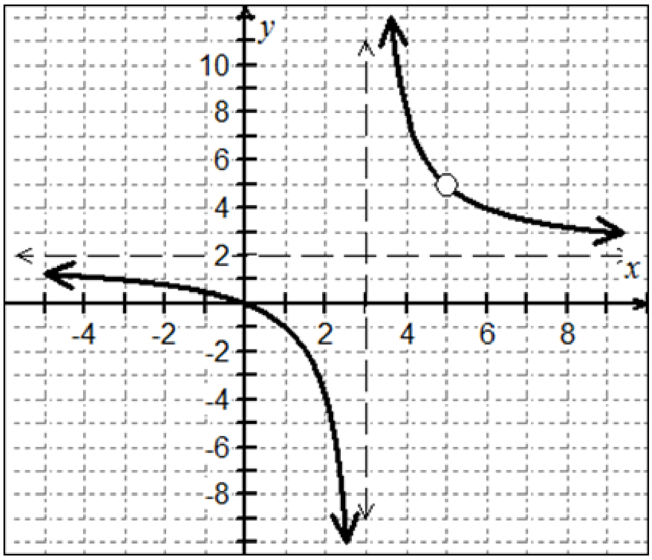
$$\lim_{x \to -\infty} \frac{x^2}{x+1} = -\infty$$

$$\lim_{x \to \infty} \frac{-x^2}{x-2} \quad \frac{(-)}{(+)} = -\infty$$

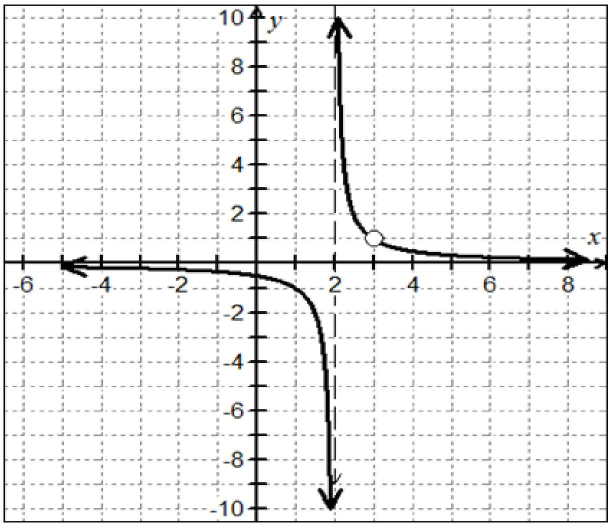
$$\lim_{x \to \infty} \frac{-x^3}{x-2} \quad \frac{(-)}{(+)} = -\infty$$

EX #6: Divide every term in the rational expression by the ***highest power of x that appears in the denominator.*** Then, apply the Properties of Limits to evaluate each "piece" to find the limit at infinity, end behavior.

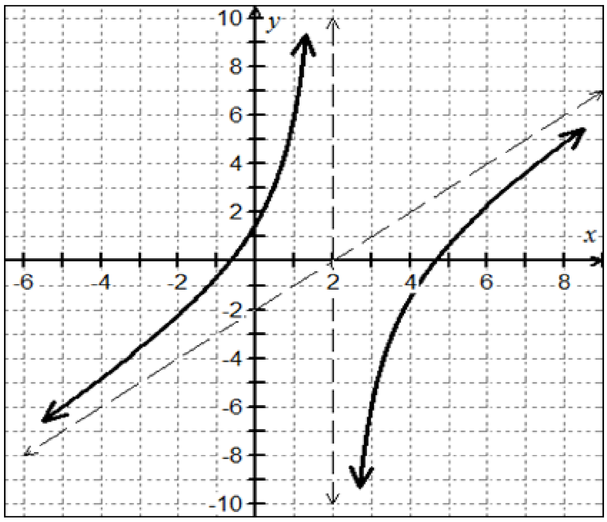
A. $\lim_{x \to \infty} \frac{2x^2 - 10x}{x^2 - 8x + 15}$



B. $\lim_{x\to\infty} \dfrac{x+3}{x^2+4x+5}$



C. $\lim_{x\to\infty} \dfrac{x^2-4x-3}{x-2}$



EX #7: Summarize and discuss the characteristics and end behavior at horizontal asymptotes and slant asymptotes based on your observations in EX #6.

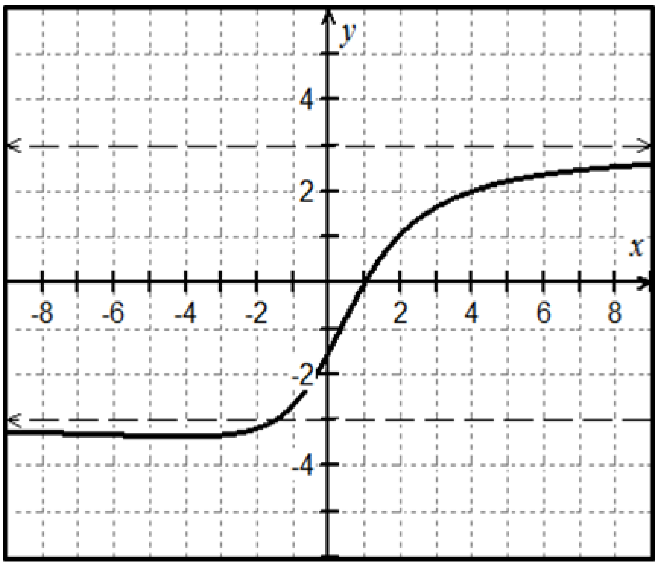
Functions with Horizontal Asymptotes

Functions with Slant Asymptotes

IN CALCULUS, YOU MUST USE NEW LANGUAGE IN ORDER TO JUSTIFY!

LIMIT DEFINITION (JUSTIFICATION) OF A HORIZONTAL ASYMPTOTE

EX #8: Use algebraic techniques to find the limits for $g(x)=\dfrac{3x-3}{\sqrt{x^2+4}}$ **, whose graph is shown.**



$$\lim_{x\to-\infty}\frac{3x-3}{\sqrt{x^2+4}}$$

$$\lim_{x\to\infty}\frac{3x-3}{\sqrt{x^2+4}}$$